The Navigability of Strong Ties: Small Worlds, Tie Strength, and Network Topology

Self-organization in Strong-tie Small Worlds

DOUGLAS R. WHITE AND MICHAEL HOUSEMAN

A small world (SW) is a (large) graph with both local clustering and, on average, short distances between nodes [1,2]. Short distances promote accessibility, whereas local clustering and redundancy of edges, as some research suggests [3,4], promotes robustness to disconnection and, through multiple independent pathways, reliable accessibility as well. For paths to transmit materials and information via network traversal, a small world also requires navigability. This was the property investigated in the first small world experiment by Travers and Milgram [5]: Could people randomly selected in Omaha, Nebraska, successfully send letters to a predetermined target in Boston, when asked to direct their letters to single acquaintances who are asked in turn to forward the letters through what becomes a chain of personal acquaintances? In many cases this task was accomplished in fewer than six steps, but success required letters sent to acquaintances who were successively closer, geographically or occupationally, to the target.

The problem of navigability is whether the next step in such chains will be any closer to the target than the last.

The problem of navigability is whether the next step in such chains will be any closer to the target than the last. This cannot occur in a network of edges generated with uniform probabilities, as Kleinberg showed [6]. SW networks with random rewiring, like random networks generally, lack the ability to find the target person quickly via successive links in the network. Kleinberg also showed a far stronger result: the ability of decentralized algorithms to find short paths by sending messages along their incident edges using only local information about them depends, in regular lattices in which edge probability is an inverse power α of lattice distance, on a unique value of α that exactly matches the dimensionality of the lattice. The short paths that are relevant in this context are those whose lengths are bounded by a polynomial in $\log N$, where N is the number of nodes, because this is what defines algorithmic efficiency for a random graph [7]. The right power-law decay of link frequency—in relation to geometric distance—creates fewer long jumps in the right direction that act as shortcuts

Douglas R. White is a mathematical anthropologist and a professor at the University of California, Irvine, where he is also Graduate Director of Social Networks and member of the Institute for Mathematical Behavioral Sciences. As part of a Traveling Scholar program of the University of Minnesota, where he received his Ph.D. in 1969, he was an NSF Cooperative Fellow at the University of Michigan in mathematical psychology and anthropology and an NIMH Predoctoral Fellow at Columbia University in anthropology and mathematical sociology. He advanced to Associate Professor at the University of Pittsburgh (1967–1979) where he co-directed the Cross-Cultural Cumulative Coding Center and took leave (1971–1973) to co-direct the National Language Study project for the Irish Republic. He is a Directeur des Etudes Invité at the Ecole des Hautes Etudes in Paris and recipient of the A. von Humboldt distinguished senior scientist prize in Germany.

to allow messages to pass quickly toward the target, whereas lower probability reduces the chance of over-jumping further away. Once in the right vicinity, honing in on a target is facilitated by more frequent short jumps in the spatial geometry. Successful search in the fewest steps is possible when the probabilities of edges vary with spatial distance so that successive edges traverse the spatial geometry at higher rates for short than for long distances. Only if the dimensional parameter of the lattice is matched exactly does a decentralized algorithm find short paths in polynomial time.

Central hubs also provide a network with searchability [8,12,54]. Hubs can also provide navigability from a global perspective, a common example being the phone book. However, as nodes or individuals with unusually large numbers of connections, hubs are lacking in social networks with strong constraints on how many links an individual may possess. Typically, social webs with such constraints have the redundancies of local clustering, and have some ties that are much stronger than the others, but can retain the SW property even with a relatively small fraction of ties that span the larger clusters. Paths of stronger ties in these networks, however, may provide access to some kinds of resources that are not accessed by weaker ties, and thus present a distinct set of problems of navigability.

Strong and Weak Ties in Small Worlds

Networks of strong ties impose constraints on how many links an individual node may possess: loosely defined, they occupy a significant portion of the limited time and energy budget. In "The Strength of Weak Ties," Granovetter showed that if a person's strong ties are those in which there is strong investment of time and affect (e.g., close friends and kin), then it is paradoxically the weaker ties that connect a person to others and to resources that are located or available through other clusters in the network [10]. In his Boston study of male professional, technical, or managerial workers who made job changes, he found that most workers found their jobs through personal contacts, but ones that were surprisingly weak: not close friends or relatives but often work-related persons and generally those with more impersonal ties with low contact frequency. Reflecting on Rapoport's information diffusion model, and Travers and Milgram's SW studies [5], he formulated his strength of weak tie hypothesis: strong ties tend to be clustered and more transitive, as are ties among those within the same clique, who are likely to have the same information about jobs and less likely to have new information passed along from distant parts of the network. Conversely, bridges between clusters in the network tend to be weak ties, and weak ties tend to have less transitivity. Hence acquaintances are more likely to pass job information than close friends, and the acquaintances of strategic importance are those whose ties serve as bridges in the network. In a generalized form of Granovetter's hypothesis, ties that are reciprocated might also tend to be more transitive than nonreciprocated ties.

Little investigation has been done on strong-tie networks in relation to small world properties. It is not surprising that with many weak ties, power-law degree distributions [11,12] or hubs, networks form small worlds in which nearly every pair of nodes is connected at relatively short distances. A feature of small worlds, like random networks [7], is that the average distance between nodes is a polynomial in log$N$, where N is the number of nodes. This is also what Barabási and Albert found for sites in the WWW where N is their network sample size [13, p 33]. What is more surprising are the results of Dodds, Muhamad, and Watts' SW experiment on 67,000 E-mail users and 18 targets in 13 countries, which estimated a true median distance to targets of six steps. People avoided asking help from others with whom they had weak ties, such as casual acquaintanceships [14]. They mostly used ties of *intermediate* strength, such as friendships formed through work or schooling affiliations. Reasons of geography were given for 50% of ties in the first three steps, and under 33% for those of work or occupation; with a reversal of these percentages was observed for steps 4–7.

Realistic Social Network Models for a Searchable Small World (SSW)

Transmission in the small world of personal networks, in the Milgram experiment, showed signs of funneling through hubs [5]. However, the Dodds et al. web replication of the SW experiments [14] shows very little reliance on hubs but rather on shared social identities; a finding corroborated by reverse small world experiments [15,16]. Watts, Dodds and Newman sought to identify a family of realistic social network models for complex small worlds with strong upper limits on how many links an individual may possess [17]. They imbue the actors in their network models with social identities. Social distance between pairs of individuals is then defined by differences in the taxonomically organized categories of identity. Like Kleinberg [6], they find that the ability to search and find specific targets depends on the network having not only short network distances, but also

Current modeling work is on large-scale longitudinal network studies of human populations and business organizations and the network dynamics of changing social, economic, and institutional configurations. His curriculum vitae and hosting of comparative longitudinal databases can be found at http://eclectic.ss.uci.edu/~drwhite; e-mail: drwhite@uci.edu. Michael Houseman, anthropologist, is Director of studies at the Ecole Pratique des Hautes Etudes (Paris) and Head of the "Systems of African Thought" research center (EPHE/CNRS, Paris). He has conducted fieldwork in Africa, Europe, and Australia and has published extensively on kinship and on ritual.

links constructed with probabilities that decay exponentially with social distance. By tuning the exponential parameter for social-distance decay of link probability, their family of models generate networks that have searchability as well as short average network distances, and they match up to describe the results of Milgram's original SW experiment.

The SSW networks of Watts, Dodds, and Newman demonstrate some further properties. Searchability increases when the hierarchies of identity are multiple rather than singular, and when these multiple identities cross-cut one another, in the sense of statistical independence [17]. This allows one step in a search to be taken on the basis of one aspect of the target's identity, whereas a next step might be taken on the basis of another aspect. Such cross-cuts move much more quickly towards the target because they move the search out of a cluster of ties in the network that reflects similarities on only one attribute (for which there may be many independent clusters) toward clusters that have many of the target's attributes and in which local ties in the cluster are more likely to lead directly to or close to the target. The introduction of multiple social dimensions leads to a much more robust result—i.e., networks are searchable for a broad range of parameters—which is very different from Kleinberg's singular condition. For navigability they require, however, not just social identities, but a network that is constructed with distance decay across the proximities defined by similarities in identities.

We offer a "navigability of strong ties" hypothesis about network topologies tested with data from kinship systems, but potentially applicable to corporate cultures and business networks.

MULTILEVEL NETWORKS

Strong-tie networks, constrained to relatively few edges per node and tending to form clusters or short cycles, seem unlikely candidates for the small world property of short average distances. Here, however, we consider networks that are *multilevel*, and we define a *system* as sets of relations among elements at different levels where each level is a graph in which each node may contain another graph structure [18]. A multilevel small world model that is of interest to us is where strong ties tend to form islands, with bridges between them, and small worlds may apply at two levels: between islands, taken as the nodes; and within islands [19]. It is of interest at both levels whether strong-tie networks have small world properties plus searchability (SSW). These properties are an important part of the story of how island clusters operate at multiple levels in many different types of networks.

Several groups of researchers have applied this multilevel approach to networks. Eckmann and Moses investigate the role of reciprocated ("strong") ties between subgraphs residing at different addresses in a network and in generating the topology and constituent units or neighborhoods of large social networks [20]. Their hypothesis applies to networks of many different sorts—neural connections in the brain (nematode), WWW links, gene regulation networks, and protein interactions—and suggests that islands of cohesive content and operational integration reside within structures that combine reciprocal links between distinct entities capable of mutual recognition and locally dense neighborhoods constructed out of these meaningful reciprocal links. They do not take networks in raw form, but take into account hierarchical links (like those in a person's Web pages) to find the reciprocal links between units that reside, so to speak, at different addresses. Thus, their means of calculating reciprocity involves indicators of mutual recognition between distinct entities. The multilevel approach means that one part of a unit subgraph may reciprocate with a different part of another unit subgraph, a crucial step in finding links of reciprocity. They test their hypothesis using a coefficient of curvature similar to the clustering coefficients of Watts and Strogatz [2,21]—measured, for example, by 3 $\times$ (number of triangles on the graph) over the number of connected ordered triples—, except that the network must be a digraph in which sending and receiving are distinguished, and the two spokes from the referent node in the connected triples must be bidirected or reciprocal edges. They found that connected triples defined by reciprocal links, consistent with the Granovetter hypothesis for strong ties [10], accounted for much of the local clustering in their social networks.

For Eckmann and Moses [20], it is crucial that their unit subgraphs are local units, such as a hierarchy of linked pages residing at the same web address and that the curvature of a node is defined purely locally as the density of ties in the immediate strong-tie neighborhood of a node. Whether or not nodes with high curvature also tend to cluster in larger neighborhoods is information gained on the topology of the network.

Islands within Islands and Measures of Cohesion

Finding the boundaries of cohesive islands or communities in a large network on a more global basis is one of the approaches we have taken to study multilevel networks [3,4], as have other researchers. Girvan and Newman, for example, calculate the betweenness centrality of each edge in a network, throw out the edge with highest betweenness, recalculate betweenness, and repeat this process for some number of edge removals [21]. The effect is to remove the bridges between clusters and to make the cohesive clusters more apparent. Because this algorithm results in a matrix of node-by-node edge betweenness, hierarchical clustering of nodes is used to identify cohesive communities and hierarchies of subclusters of greater cohesion within the larger communities. The results of this algorithm give encouraging results when edge removals are stopped when each node has more within-cluster than between-cluster edges.

This procedure is consistent with the graph theoretic concept of multiconnectivity (node connectivity as distinct from the degree connectivity of a node) in which the following two definitions are equivalent (Menger's Theorem): A subgraph S of a graph G is *k-connected* (1) if and only if it has at least k+1 nodes and is not separable by removal of fewer than k nodes, and (2) if and only if it has at least k+1 nodes and there exist at least k paths between every ordered pair of nodes with no intermediate nodes in common, i.e., node-independent paths. A maximal k-connected subgraph S of a graph G is called a *k-component*, or for successive values of k, a component, bi-component, tri-component, 4-component, and so forth [4,22]. These components are, by definition, hierarchically stacked, but they may also overlap. To avoid confusion with these terms it is important to specify k-components with $N \geq k$ nodes because algorithmic computer science definitions of the same terms drop the requirement that a k-component has at least $k+1$ nodes, in which case Menger's Theorem no longer applies.

Powell et al. model the industry network of contractual biotechnology collaborations from 1988 to 1999 in relation to firm-level organizational and financial changes [23]. The industry is knowledge-based, with extensive reliance on organizational learning that occurs through networks of dense collaborative ties among organizations. Firms show a power-law distribution of degree connectivity. There are several kinds of stochastic processes (contagion, heterogeneous mixing of Poisson processes) that can generate such power-law distributions [24]: This one is not preferential attachment or a simple popularity bias [13] because conditional logit estimates of factors that might influence the distribution show that attachment to those firms with shared multiconnectivity has a very strong effect ($p < 10^{-9}$), followed by partner's diversity of ties; and, controlling for these, partners with fewer ties and less experience have a smaller effect on attachment.

In testing the hypothesis that k-connected cohesion has significant effects, Moody and White [3] found common k-connectivity in director interlock business networks [25] to be a strong predictor of similarity in political contributions, controlling for a host of other network variables and firm attributes. For adolescent friendship groups in a replication study across 12 large high school networks, they found k-connectivity to be the best predictor, with a common slope, of an independent multi-item attitudinal measure of school attachment even when controlling for other network and school variables as well as individual attributes.

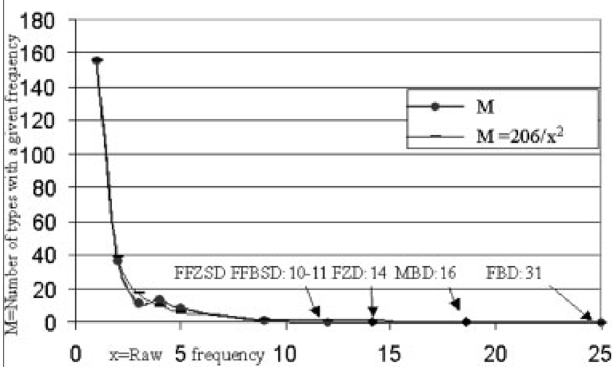
Networks of strong ties impose constraints on how many links an individual node may possess: loosely defined, they occupy a significant portion of the limited time and energy budget.

The concept of cohesion—cohesive subsets—plays a fundamental role in the study of kinship networks, where the primary relations of parent/child and husband/wife are strong ties, and large islands of subgroup cohesion are formed by endogamy. Structural endogamy [26] is defined as a bi-component on a kinship graph with married couples and unmarried individuals as nodes and parent-child links as edges. The algorithm for finding bi-components in a network is a depth-first search through a spanning tree that identifies maximal sets of cycles that share edges, with processing time of order $O(e)$, proportional to the number of edges. Brudner and White exploited this ability to find islands of endogamy in networks of any size to test the hypothesis that among Carinthian farming populations, the class stratification distinction of principal farmstead heirs and their spouses from siblings who receive minor inheritance was strongly predicted from bi-component membership [27]. Similarly, in Johanson and White's study of Turkish pastoral nomads that exports its population surplus to villages, bi-component membership is an almost perfect predictor of stayers versus leavers [28, 29]. Houseman, after showing the well defined bi-component memberships for Indigenous Australian groups, argued that they could solve many of the problems in determining land entitlements since they identify those who have consistently intermarried within each group [30].

KINSHIP NETWORKS AND COMPLEXITY

Kinship networks provide a means of studying large-scale networks composed of strong ties. Hubs are not relevant aspects of a network model for kinship because few societies have "fat tail" outliers in terms of power laws for number of children, the exception being historical societies with harems or concubines [31]. Derrida et al. show that with random marriage and reasonable Poisson-distributed numbers of children in a steady-state population of tens of thousands of people, in 15 generations about 80% of the founders appear in the genealogical tree of every individual, and the 20% that do not appear are those who have left no descendants, so that most pairs of adults have nearly all their root ancestors in common [32]. Common single ancestors appear much earlier for pairs of descendants. Up to 9 generations, the distribution of number of common ancestors for pairs of people follows an exponential distribution skewed—as expected from expected child-edges that are equiprobable, and differences in degree connectivity

FIGURE 1



Power-law distribution of frequencies of marriage types. F, father; M, mother; Z, sister; B, brother; S, son; D, daughter; e.g., FBD is father's brother's daughter marriage.

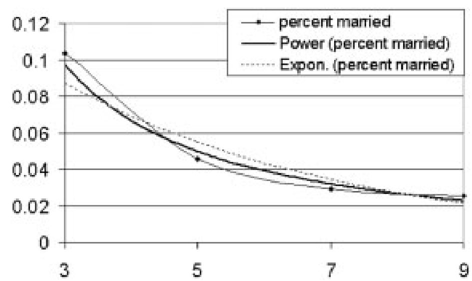
cumulate exponentially—toward decreasingly fewer common ancestors for increasingly many people.

Our first experiments tested the connections among kinship, small world properties in strong-tie networks, and indicators of network complexity in modeling networks of an Arabized society of pastoral nomads in South Turkey [28,29]. Ethnographer Ulla Johanson provided excellent genealogical ancestries for five generations, and some living members remembered deeper ancestries of founders, for a total network of about 1100 people who were members of the same clan, plus another 200 or so outsiders (villagers or members of other nomad clans or tribes) with whom they had married. For a society with 5–9 known generations such as the nomads, it was no surprise, given Derrida's simulations [32], to find an exponential distribution of ancestors by number of descendants. This was a society, like many in the Middle East with Semitic and especially Arab roots—or heavily influenced by Arabization and Islam—with strong endogamy, segmented deep patrilineages, both lineage-endogamous and lineage-exogamous marriages, a preference for men to marry with a father's brother's daughter (FBD), a close lineage mate, and also a staggeringly diverse array of marriages with other consanguines (blood relatives of 234 distinct types for two spouses having a common ancestor back seven generations or less). The first test, with results shown in Figure 1, was whether the distribution of marriage frequencies by type of kin married was exponential, as expected from equiprobable marriages within ego's generation, Poisson—as might be expected from a single norm for preferred type of partner type—or power law, possibly consistent with an SSW model. The x axis here is the number of spouses related by each of the 234 types of blood kinship, and the y axis the number M of types with that many related spouses. The distribution is a power law that follows an inverse square. The high-frequency outlier in this distribution is FBD marriage, nearly twice the raw frequency (31, binned in its location on the graph) of the next rival.

Our "navigability of strong ties" hypothesis about network topologies comes out of wondering whether power laws of this sort are suggestive of self-organizing properties in segmented-lineage systems of the Middle East. Before our study began, White and Johansen [29] had used Eckmann and Moses' approach to curvature in considering reciprocal marriage links between different parts of the hierarchically embedded sublineages within the 10 maximal patrilineages. Each of these has a meaningful social identity and marriages are often arranged between them. Strong ties of reciprocal marriage alliances between sublineages—ones that open the door to potential brides as part of the extended relationships of trust and mutual visiting—had the structural properties of small worlds: a high average node-specific clustering coefficient, a triad census [33] that fit a larger-scale clustering model, but also much higher rates of intransitive chains of reciprocal ties that bridged different clusters, which gave the strong-tie network a very low average distance between sublineages. Here, in a sociological as well as statistical sense, the significant bridges were strong ties. An overall scaling of nodes at this level in the multilevel network (from individuals, to couples, to minimum and maximum lineages) showed a nearly one-dimensional alignment of maximal lineages along a remoteness-from-villages continuum, but with significant second-dimension variability for specified sublineages who departed from the alliance patterns of their maximal lineage [29, Chapter 5].

These two suggestive results led us to investigate more fully a small world model for strong-tie networks, this time focusing on Kleinberg's searchability parameter [6]. The hypothesis was that the full range of types of consanguineal marriage (as in Figure 1) would fit Kleinberg's model for a small world with searchability where the parameter (α) describes a power-law decay of the probability of a marriage link as a function of kinship distance. The result of this test is presented in Figure 2 for the distribution of the percentages of marriages with cousins at different genealogical distances and shows a preference gradient for closer cousins. Here the kinship distances through a parent (P), child (C) or sibling (~), at the same-generation of cousins, fall along a gradient from three for first cousins (P~C), five for second cousins (PP~CC), seven for third cousins (PPP~CCC), and nine for fourth cousins (PPPP~CCCC). The probability of marrying a cousin as indexed by the percentages of available cousins married at each distance declines inversely to kinship distance to the power 1.6. This supported the idea, given that marriage links at the sublineage level are highly clustered, that the nomad clan multilevel network is a searchable small world, because the average of marriage distances are low. Further, the scaling dimensionality of the marriage network was known from

FIGURE 2



Cousin marriage probability estimates (rates per available) fit to small world searchability power law $1/d^{\alpha}$, where d is distance and $\alpha = 1.6$ power.

the curvature modeling to be between 1 and 2, so the power-law slope of 1.6 is consistent with Kleinberg's optimal criteria for searchability [6]. The fit to a power-law distribution, shown by the solid line in Figure 2, is closer than the fit to an exponential distribution (dotted line).

These findings led White and Johansen to investigate the historical and comparative data on segmented lineages with FBD marriage [29]. A study by Korotayev [34] proved consistent with other evidence in convincing them that the features of Turkish nomad segmented lineages are very widely distributed among Semitic and Arab-influenced societies structured along similar lines, in spite of differences in group and network size. We conceive of the elements of this social organizational complex as a self-organizing segmentary system with a dynamic of small world cohesion through gradients of endogamous marriage.

We conceive of the elements of this social organizational complex as a self-organizing segmentary system with a dynamic of small-world cohesion through gradients of endogamous marriage. . . . where women are dynamic agents.

It is also clear from the combined sources that societies with this segmentary system emerged in a broad-scale network in the Middle East linked through international exchange, maritime and camel trade, ones that formed part of a large civilizational complex. In these interconnected societies the corporate organization was that of extended patrilineages with distributed members capable of engaging in trade. A basic social unit was the patrilocal family that formed part of larger patrilineages in which some members were dispersed along trade routes and many were engaged in a local subsistence component of pastoralism. While not exclusively Arab, this system diffused and extended itself very widely with the conquests of the Arab Caliphates of the 7th and 8th centuries [34].

The other element that suggests self-organization stems from the fact that the co-residential lineage unit in many of the contexts in which segmented-lineage systems operate was the relatively small extended patrilineal family. If each family in each generation has two or three incoming women as wives and a comparable number of daughters exiting as wives for other groups, this number of links in a total network of *N* families is barely enough to maintain the biconnectivity needed to integrate the lineages into an exchange system. For such a system to work, with wives as the intermediary link in marriage alliances, it is necessary for women to retain their membership and rights (e.g., inheritance) in their natal group, which would help explain the emphasis on patrilineal corporations, and hence on a multilevel network of individuals and corporations. But with scarce links as precious assets, and a network on the verge of falling apart into components that lack cohesive integration through marriages, how links are distributed becomes a crucial issue, in Bak's phraseology, for self-organized criticality of multi-connectivity [9].

The study of network topology using the Eckmann and Moses curvature methods showed how an optimal small world with searchability (SSW) would follow from self-organizing locally based or distributed mechanisms. If consolidation of alliance and trust is established or signaled through reciprocal exchanges of brides between sublineages, which is a common feature of these systems, creating strong ties as channels for visiting and familiarity, and these strong ties are organized as small worlds, they provide the mechanism for marriage contacts dependent on visiting and intimate knowledge of other families in a distance-day distribution which, when transformed into actual marriages, recreates the distance-decay power slope (the α of Figure 2). The bridges here are strong ties, not weak. Local behavior generates the network topology in which further local behaviors, like the search for allies or particular types of exchange partners, can operate through channels of trust.

This networked system was developed on a regional scale only once in the history of civilizations, but once in place, searchability is a feature of the global network topology generated by the local behavior. This kind of model of self-organization in kinship behavior would help to explain how the segmented-lineage form of organization is able to mobilize in response to conflict at a segmentary level. Networks of strong ties of trust between smaller units, highly clustered, but connected in a searchable small world topology, given the smallest conflict, or a conflict that escalates to any level, facilitates mobilization into opposing factions that are quite predictable and self generating given the fault lines among the segmented units, which are hierarchical. As witnessed in Afghanistan, these fault lines are also highly malleable, given that at the local level the stronger alliances change as a function of the slower tempo of marriages, but at the more global level new enemies may present themselves from the outside and may mobilize whole populations and regions.

The rough outlines of this understanding of self-organizing multiple-level networks in the Middle East has long been known to anthropology in the form of a segmentary lineage theory that only needs to be generalized to include a range of variants, such as variations in lineage depth, whether segments are also spatially organized, size of lineages, type of subsistence, variants in rules of exchange, etc. The new element added by models of networks and complexity, however, is to consider the dynamic element of new marriages that continually re-sculpt the contours of cohesion through bi-connectivity, and that act as signals, tentatives, and potentially reciprocal enactments of consolidated trust between allied lineage segments. The system is not simply one organized by hereditary links among men. Women are dynamic agents, and their memberships in their natal-kinship corporations, with full rights, are vital to the dynamics that sustain the internal moral and material economies of segmented-lineage systems. Such is the conclusion, at any rate, of White and Johansen [29]. To clinch the Middle Eastern example, the cross-cutting *identities* of nodes in segmented lineages match in type with those of the social identity SSW models [17], where searches based on multiple and cross-cutting identities such as occupations and locations help to find shortcuts that bring a search closer to a target. Lineage segments and affines play this role in Middle Eastern segmented-lineage systems.

Extending the Investigation of Strong-tie Kinship Networks

As collaborative researchers on strong-tie kinship networks [35–37], our joint interest in kinship networks and issues of complexity (self-organization, criticality, multilevel networks, and exponential versus power-law signatures of potential organizing processes, etc.) was piqued by White and Johansen's recent findings [29]. We quickly mobilized to analyze as many of our existing kinship databases as we could to explore the signatures of frequency distributions for types of consanguineal marriages and affinal relinkings, which is the anthropological term for a marriage that is not between blood kin but that reconnects blood-kin groups ("families") already connected through marriage.

In a restudy of 10 societies, we looked at distributions similar to that of Figure 1, but now for consanguineal marriages and for two- and three-family relinkings, using the GENOS program [38]. Results quickly formed a pattern. In those societies with a high proportion of consanguineal marriages, these tended to form power-law distributions, whereas the relinkings fit a distribution expected from marriage types that are equiprobable, where differences cumulate exponentially, except for a handful of preferred relinkings with higher than expected frequencies, such as sister exchanges. This was true for the White-Johansen Turkish nomads [29], for the two Amazonian societies, the Arawete and the Parakana [39,40], the Indian Chenchu hunter-gatherers [41], and a "remote" Australian Aboriginal group, the Alyawarra [42,43]. In the other set of five societies having little or no consanguineal marriages, it is the two-family relinkings—but not those of three families—that tended to form power-law distributions. These cases included two European societies, the Tory Islanders of Ireland [44] and the Feistritz villagers of Carinthia, Austria [27], and three "settled" Australian aboriginal populations: the Yaraldi, Nyungar, and Wilcania villagers [45–47].

The first set of societies has kinship networks that are explicitly multileveled. Among the Turkish nomads, consanguines are organized into discrete, segmented lineages that allocate wealth assets to members. In the Amazonian and Indian cases, kinship ties are classified terminologically according to a strict bipartite principle in which consanguines and affines are opposed as nonoverlapping sets of relations. In the "remote" Australian case, there are two such overarching categorical divisions that are cross-cutting. This of course does not exhaust the various ways in which kinship networks may be explicitly organized in a multileveled fashion. The five societies considered here allow close consanguineal marriage. Others societies (often termed "semi-complex"), while possessing kinship-based corporations, prohibit close kin marriage; in such cases, the kinship network seems to be structured through the regular reiteration of alliances between *distant* rather than close blood-kin [48,49].

Our tentative hypothesis is that societies with such explicit, multilevel kinship networks—incorporating kinship corporations and/or classificatory associations that allocate statuses—tend to pursue strategic alliances in the construction of SSW network topologies through arrangement of marriages between consanguines, hence the characteristic power-law distributions. In the second set of societies, in which kinship corporations or wide-ranging classificatory schemes are lacking, the kinship network is egocentric. However, the presence of power-law distributions for relinkings of two families—but not three—suggests that, in such cases, it is these relinkings, rather than consaguineal marriages, that provide the basis for strategic alliances in the construction of SSW network topologies. From this point of view, these latter kinship networks may be held to be multileveled in an implicit rather than explicit fashion. Preliminary analysis of the average coefficient of the power-law distributions of relinking in this second set of societies shows a relationship that is perfectly consistent with Kleinberg's SSW network topologies [6] for spatially distributed families whose likelihood of two-family relinkings is an inverse square function. Our "navigability of strong ties" hypothesis regarding network topologies is that strong ties, in addition to being clustered, often give unique access to valued resources; moreover, there are some conditions and tipping points at which issues of uncertainty in survival make broader strong-tie accessibility to particular resources co-evolve with strong-tie SSW network topologies. These typologies solve, independently of weaker ties, both the small distance and the searchability problems. In general, strong-tie SSW network dynamics and architectures need to be investigated as part of the self-organizing morphogenetics of how cultures reproduce themselves, and at multiple levels: kinship as self-reproducing networks [29], and corporate cultures and business networks in self-sustaining economies [50] are prime examples, among others [51]. Lévi-Strauss's exchange theory provides a powerful framework for the analysis of consanguineal marriages but the structural principles governing relinking have yet to be understood, and his conception of marriage rules falls short in dealing with the organization of empirical diversity. The power-law distributions of relinkings in our group 2 societies (that Lévi-Strauss could call "complex") suggest a place to begin. On a more general level, the fractal structure of the internal diversity of consanguineal and relinking marriages even in "classical" cases of Australian kinship [42,43], usually analyzed by means of a simple mechanical model, requires a reworking of exchange and alliance theory, bringing segmented-lineage organization as well into this reworked framework [29,52]. This is work in progress, and in the next stage of our research the kinship distance for every marriage, i.e., the shortest cycles in our graphs, will be computed so as to have the analogues to both Figures 1 and 2 for each case and for marital relinkings of two and three families as well as consanguineal marriages. For that we need one of Mark Newman's famously fast algorithms to apply to our database of large-scale networks.

In business [23,25] and kinship organization . . . cohesiveness measured by multiconnectivity [4] proves to be a component of searchable strong-tie small worlds that are self-organizing systems, complete with network criticalities, signatures of fractality, curvatures that lend themselves to topological analysis, and distance-decay parameters that define the searchability of small worlds.

DISCUSSION

Questions about structural bias in networks, such as reciprocity (a feature of strong ties), or transitivity (a common side effect of strong ties), led network researchers like Rapoport [53], Milgram [5], Granovetter [10], and Watts [1] to investigate how even very large communities are still small worlds in which networks may be strongly clustered, yet no two people—or very few—are much further away in the network, for example, than six degrees of separation. Rapoport, for example, showed that rumors followed a logistic or lazy *S*-shaped curve in their spread through a high school typical of a diffusion process [53]: In the first few steps of transmission they spread to very few people overall; with a few more steps they spread like wildfire through the school; but oddly, it seemed, they never reached a sizeable residual set of people who seemed to be insulated (a comparable random network, in contrast, usually reaches everyone). As did Rapoport, Granovetter showed the first stage to correspond to spread within cliques, the second to spread between cliques, and the third stage, of a barrier to further spread, to correspond to cliques or isolates that do not link to the larger network.

Research on SWs advanced quickly after Watts and Strogatz showed the robustness and ubiquity of SW networks in their two parameter model of clustering and distance [2]. The modeling of scale-free power laws and preferential attachment begun by de Sola Price [11,12] and Rapoport [53] and continued in the stochastic processes literature [24], missing both in the Erdös-Renyi random graph model [7] and the Watts-Strogatz rewiring models [2], was reintroduced with great success by Barabási and Albert [54]. Kleinberg [6] discovered the effect of network topology and appropriate parameters for searchability, and more realistic social identity small world modeling [17] identified network structures that are searchable for a broader range of parameters. We have not reviewed here all the work done on the effects of timing in the entry and exit of nodes on degree connectivity and network topology, but we have focused instead on the relationship between strength of ties and small worlds.

Within our own areas of research, concepts of network cohesion, parametric properties of clusterability, average distance and topological searchability gradients in networks have proven important in understanding multilevel networks with individuals and higher order nodes such as corporate groups, both in business [23,25] and kinship organization. Cohesiveness measured by multiconnectivity [4] proves to be a component of self-organizing systems, complete with network criticalities, potential signatures of fractality in power-law distributions, measures such as curvatures that lend themselves to topological analysis and distance-decay parameters that define the searchability of small worlds. One of the crucial insights both from kinship studies and from business networks such as the biotechnology industry is that multilevel network phenomena, including the hierarchies of cohesive subgroups in networks, are critically related to issues of self-organization. Another is that the investigation of small worlds composed of strong ties and topologically structured by searchability criteria are among the multilevel network phenomena that need investigation. They provide important clues to the functional autonomy of many types of community-like and cooperative or collaborative organizational structures, as well as the types of segmented-lineage systems capable of rapid mobilization in the escalation of conflicts, as in the Middle East.

Open research problems, in assessing distinctions and findings in this area, include further study of the small world and community network topologies and their social effects, how the strength or weakness of ties in Granovetter's sense are related to them, and how to integrate new findings in relation to broader social theory. Among the interesting open questions are those that derive from graph theory distinctions between bridge ties as edge-cuts and individuals as node-cuts whose removal disconnects a network [4], and how these play out in networks constructed either by dyadic links, intersections of memberships in groups [17], or intersections of memberships plus dyadic links between members in different [55]. Is it the bundles of bridging ties or the sets of individuals with multiple memberships that do the contracting in a small world or community network? Are individuals or ties the binding elements in social cohesion and the bridging elements in social worlds?—or does this question require some of the ways in which network theories have excelled in understanding the interdependencies or dualities that exist between the different aspects of the question? Raising perspectives such as these in the context of network research was one of the original contributions of Granovetter's strength of weak ties argument. Hopefully the new work on navigability by Kleinberg [6] and Watts, Dodds, and Newman [17], and tests of hypothesis such as the "navigability of strong ties," will continue to stimulate new research.

ACKNOWLEDGEMENTS

This research was supported by NSF grant No. 9978282. Doug White is indebted to John Padgett for the invitation to join the Working Group on the Co-Evolution of States and Markets at SFI, and to the editors of the Encyclopaedia of Community, who permitted some of the literature review presented here to be reused. Mark Granovetter and Duncan Watts provided extensive feedback here as well as on the discussion of literature prepared for the Encyclopaedia entry. Jim Crutchfield, Jon Kleinberg, Mark Newman, Peter Bearman, Harrison White, Richard Alba, Ryan Kernan, and Jean-Pierre Eckmann offered amendments on the text; and much of this literature was discussed with, and some found by, Scott White. Padgett's commentary, as on a related article by Powell, White, Koput, and Owen-Smith, has been consistently helpful, as have discussions with researchers at the Santa Fe Institute, too many to name individually, but SFI support is appreciated for participation in the Working Groups on the Co-Evolution of States and Markets organized by Padgett and on Networks and Complexity, organized by Crutchfield and Watts. We thank Laurent Barry for use of his GENOS software for analyzing relinkings. Other analyses used PGRAPH software (Par-Calc) discussed elsewhere by White.

REFERENCES

1. Watts, D.J. Small Worlds: The Dynamics of Networks Between Order and Randomness; Princeton University Press: Princeton, NJ, 1999.
2. Watts, D.J.; Strogatz, S. Collective Dynamics of "Small-World" Networks. Nature 1998, 393, 440–442.
3. Moody, J.; White, D.R. Structural Cohesion and Embeddedness: A Hierarchical Conception of Social Groups. American Sociological Review, forthcoming.
4. White, D.R.; Harary, F. The cohesiveness of blocks in social networks: Connectivity and conditional density. Sociological Methodology 2001, 31, 305–359. (Blackwell Publishers: Boston, 2001.)
5. Travers, J.; Milgram, S. An experimental study of the small-world problem. Sociometry 1969, 32, 425–443.
6. Kleinberg, J. Navigation in a small world. Nature 2000, 406, 845.
7. Erdös, P.; Renyi, A. On the evolution of random graphs. Magyar Tud Akad Mat Kut Int Kzl 1960, 5, 17–61.
8. Adamic, L.A. The Small World Web. Proceedings of the 3rd European Conference on Research and Advanced Technology for Digital Libraries, Paris, 1999, pp 443–452.
9. Bak, P.; Tang, C..; Wiesenfeld, K. Self-organized criticality: An explanation for 1/f noise. Phys Rev Lett 1987, 59, 381–384.
10. Granovetter, M. The strength of weak ties. Am J Sociol 1973, 78, 1360–1380.
11. de Solla Price, D.J. Networks of scientific papers. Science 1965, 149, 510–515.
12. de Solla Price, D.J. A general theory of bibliometrics and other cumulative advantage processes. J Am Soc Inform Sci 1980, 27, 292–306.
13. Barabási, A.-L. Linked: The New Science of Networks; Perseus Publishing: Cambridge, MA, 2002.
14. Dodds, P.S.; Muhamad, R.; Watts, D.J. An experimental study of social search and the small world problem. Manuscript submitted. 2002.
15. Russell, B.H.; Killworth, P.D.; Evans, M. J.; McCarty, C.; Shelly, G.A. Studying relations cross-culturally. Ethnology 1988, 27, 155–179.
16. Killworth, P.D.; Bernard, H.R. The reverse small world experiment. Social Networks 1978, 1, 159–192.
17. Watts, D.J.; Dodds, P.S.; Newman, M.E.J. Identity and Search in Social Networks. Science 2002, 296, 1302–1305.
18. Harary, F.; Batell, M. What is a system? Social Networks 1981, 3, 29–40.
19. Boorman, S.A.; Levitt, P. R. The Genetics of Altruism; New York: Academic Press.
20. Eckmann, J.-P.; Moses, E. Curvature of co-links uncovers hidden thematic layers in the World Wide Web. Proc Natl Acad Sci USA 2002, 99, 5825–5829.

21. Girvan, M.; Newman, M.E.J. Community structure in social and biological networks. Proc Natl Acad Sci USA 2002, 99, 8271–8276.
22. Harary, F. Graph Theory; Addison-Wesley: Reading, MA, 1969.
23. Powell, W.W.; White, D.R.; Koput, K.W.; Owen-Smith, J. Practicing Polygamy with Good Taste: The Evolution of Interorganizational Collaboration in the Life Sciences. Manuscript, 2002.
24. Bartholomew, D.J. Stochastic Models for Social Processes, 2nd Ed.; Wiley: New York, NY, 1967.
25. Mizruchi, M.S. The Structure of Corporate Political Action; Harvard University Press: Cambridge, MA, 1992.
26. White, D.R. Structural Endogamy and the Graphe de Parenté. Informatique, Mathématique et Sciences Humaines 1997, 137, 107–125.
27. Brudner, L.A.; White, D.R. Class, property and structural endogamy: Visualizing networked histories. Theory and Society 1997, 25, 161–208.
28. Johansen, U.C.; White, D.R. Collaborative long-term ethnography study and longitudinal social analysis of a nomadic clan in SoutheasternTurkey. In: Chronicling Cultures: Long-Term Field Research in Anthropology; Kemper, R., Peterson Royce, A., Eds.; AltaMira Press: Walnut Creek, CA, 2002, pp 81–99.
29. White, D.R.; Johansen, U.C. Network Analysis and Ethnographic Problems: Process Models of a Turkish Nomad Clan; Lexington Press: Lexington, MA, Forthcoming.
30. Houseman, M. Marriage networks among Australian Aboriginal Populations. Australian Aboriginal Studies 1997, 2, 2–21
31. Betzig, L.L. Despotism and Differential Reproduction: A Darwinian View of History; Aldine de Gruyter: New York, 1986.
32. Derrida, B.; Manrubia, S.C.; Zanette, D.H. Statistical properties of genealogical trees. Phys Rev Lett 1999, 82, 1987–1990.
33. Batagelj, V.; Mrvar, A. A subquadratic triad census algorithm for large sparse networks with small maximum degree. Social Networks 2001, 23, 237–243.
34. Korotayev, A. Parallel cousin (FBD) marriage, Islamization, and Arabization. Ethnology 2000, 39, 373–395.
35. Houseman, M.; White, D.R. Structures réticulaires de la pratique matrimoniale. L'Homme 1996, 139, 59–85.
36. Houseman, M.; White, D.R. Network mediation of exchange structures: Ambilateral sidedness and property flows in Pul Eliya, Sri Lanka. In: Kinship, Networks and Exchange; Schweizer, T., White, D.R., Eds.; Cambridge University Press: New York, 1998, pp 59–89.
37. Houseman, M.; White, D.R. Taking sides: Marriage networks and dravidian Kinship in lowland South America. In: Transformations of Kinship; Godelier, M.; Trautmann, T., Tjon Sie Fat, F., Eds.; Smithsonian Institution Press: Washington, DC, 1998, pp 214–243.
38. Barry, L.S. Les modes de composition de l'alliance. Le mariage 'arabe.' L'Homme 1998, 147, 17–50.
39. Viveiros de Castro, Eduardo. From the Enemy's Point of View : Humanity and Divinity in an Amazonian Society; Chicago University Press: Chicago, 1992.
40. Fausto, C. A dialectica da predacão e da familiarizacão entre os Parakana da Amazonia Oriental. Por uma teoria da guerra amerindia. Doctoral thesis, PPGAS/Museu Nacional/UFRJ. 1997.
41. Furer-Heimendorf, C. The Chenchus; Macmillan: London, 1943.
42. Denham, W.W.; McDaniel, C.K.; Atkins, J.R. Aranda and Alyawarra Kinship: A quantitative argument for a double helix model. American Ethnologist 1979, 6, 1–24.
43. Denham, W.W.; White, D.R. Sided with Omaha and a Twist: The Three Logics of Alyawarra Kinship. Paper presented in conjunction with the American Anthropological Association meetings, New Orleans, 2002.
44. Fox, R. The Tory Islanders; Cambridge University Press: Cambridge, 1978.
45. Berndt, R.M.; Berndt, C.H. A World that Was. The Yaraldi of the Murry River and the Lakes, South Australia. University of British Colombia Press: Vancouver, 1993.
46. Tilbrook, L. Nyungar tradition: Glimpses of South-western Australia 1829–1914. University of Western Australia Press: Nedlands, 1983.
47. Memmott, P. Humpy, House and Tin Shed: Aboriginal Settlement History on the Darling River; University of Sydney Press: Sydney, 1991.
48. Héritier, F. L'Exercice de la parenté. Seuil/Gallimard: Paris, 1981.
49. Héritier-Augé, F.; Copet-Rougier, E. Les complexités de l'alliance. Vol. I : Les systèmes semi-complexes; Archives Contemporaines: Paris, 1990.
50. White, H.C. Markets from Networks: Socioeconomic Models of Production; Princeton University Press: Princeton, NJ, 2002.
51. Bian, Y. Getting a Job through a Web of *Guanxi* in Urban China. In: Networks in the Global Village; Wellman, B., Ed.; Westview: Boulder, CO, 1999.
52. Houseman, M. Les structures de l'alliance chez les Beti: Analyse critique du fonctionnement matrimonial dans les systèmes semi-complexes. In: Les complexités de l'alliance. Vol. I : Les systémes semi-complexes. Héritier-Augé, F., Copet-Rougier, E. ; Ed.; Archives: Paris, 1990.
53. Rapoport, A.A. Contribution to the theory of random and biased nets. Bull Math Biophys 1957, 19, 257–271.
54. Barabási, A.-L.; Albert, R. Emergence of scaling in random networks. Science 1999, 286, 509–512.
55. Mische, A.; Pattison, P.E. Composing a civic arena: Publics, projects and social settings. Poetics 2000, 27, 163–194.